From Michael Fried's Introduction to Conics IV

In reading and translating Book IV, I have tried to give Apollonius a fair chance, to keep modern algebraic ideas about conics at a distance, and to view the text with eyes trained only on the mathematical and philosophical concerns of Apollonius's contemporaries and on the geometrical character of the previous three books of the *Conics.* Approaching the text this way allows one to see that Book IV, far from being dull, reveals fundamental difficulties in Apollonius's treatment of conic sections. First and foremost of these is, of course, the troubling nature of the opposite sections. But besides that, the book also raises questions as to Apollonius's basic understanding of how conic sections may be present and related to one another in a single plane and this understanding is crucial in reading the whole of the *Conics.*

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The truth is, both in the letter introducing Book I, which also introduces the whole of the *Conics,* and in the letter introducing Book IV itself, Apollonius tells us quite plainly what Book IV is about: "This book treats of the greatest number of points at which sections of a cone can meet one another or meet a circumference of a circle...and, moreover, the greatest number of points at which a section of a cone or a circumference of a circle can meet opposite sections. Besides these questions, there are more than a few others of a similar character." Among the these "few others," clearly, are all those concerning the greatest number of points opposite sections can meet opposite sections (that this is one of the principal subjects of Book IV is said explicitly in the introduction to Book I). Now, Apollonius reports that Conon of Samos treated the case where a conic section or circumference of a circle meets another conic section, but that Conon's demonstrations were incorrect. Apollonius further reports that, in connection with Conon's flawed proofs, Nicoteles remarked that the case in which a conic section meets opposite sections could be solved, but, as Apollonius makes sure to say, neither Nicoteles nor anyone else provided a demonstration. As for the greatest number of points in which opposite sections can meet opposite sections, Apollonius says that no one has ever noticed this question, let alone treated it. Thus, as in the introduction to the first book, Apollonius promises his readers that in this book they can expect a fuller and more rigorous treatment (and, therefore, to his mind, a more *correct* treatment) of familiar questions, but also completely new material, which, as in so much of the *Conics,* is precisely that concerning the opposite sections.

The importance of the opposite sections in the *Conics* cannot be overemphasized. The existence of opposite sections may have been known before Apollonius, as the references to Conon and Nicoteles in Apollonius's prefatory letter to Book IV suggest, however, it is highly doubtful that, before the *Conics,* there was anywhere a more than a perfunctory treatment of them. The opposite sections are peculiar, and, in the enunciations to propositions in the Conics, Apollonius usually separates them from the other conic sections. This peculiarity, in part, has to do with their number, for while there is a sense in which the opposite sections are one curve, as a visual object they are clearly two. Thus, for one, like Apollonius, whose work with curves is always governed by a fundamentally geometric outlook, the plural-singular nature of the opposite sections makes them an object of fascination, but it also duly gives rise to a certain uneasiness with them, which one senses already in the first book of the *Conics.*