CHAPTER IX.

GENERAL EQUATIONS OF THE ELECTROMAGNETIC FIELD.

604.] In our theoretical discussion of electrodynamics we began by assuming that a system of circuits carrying electric currents is a dynamical system, in which the currents may be regarded as velocities, and in which the coordinates corresponding to these velocities do not themselves appear in the equations. It follows from this that the kinetic energy of the system, in so far as it depends on the currents, is a homogeneous quadratic function of the currents, in which the coefficients depend only on the form and relative position of the circuits. Assuming these coefficients to be known, by experiment or otherwise, we deduced, by purely dynamical reasoning, the laws of the induction of currents, and of electromagnetic attraction. In this investigation we introduced the conceptions of the electrokinetic energy of a system of currents, of the electromagnetic momentum of a circuit, and of the mutual potential of two circuits.

604.1 We then proceeded to explore the field by means of various configurations of the secondary circuit, and were thus led to the conception of a vector $\mathfrak{A}$, having a determinate magnitude and direction at any

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604. *In our theoretical discussion of electrodynamics we began…*: Maxwell's "theoretical" discussion began with Chapter VI of Part IV. Earlier stages of the *Treatise*, in contrast, were devoted largely to the acquisition of electrodynamic concepts. In the first few Articles of this chapter Maxwell will review selected stages in the mathematical construction of the theory.

a homogeneous quadratic function of the currents…: See, for example, (571.4).

purely dynamical reasoning: Here, fitting the general equations of Chapter V to the special conditions of electrical circuits and fields—as opposed to deducing consequences from hypotheses. The role of experiment differs dramatically in the two methods; Maxwell's luminous characterization of the way experiment is to function in dynamical reasoning (592.2) will be amplified further in (606.) below.

604.1 *the conception of a vector* $\mathfrak{A}$: As Maxwell is about to remind us, he first introduced the vector $\mathfrak{A}$ as the "vector potential of magnetic induction" in Art. 405; he rediscovered it through dynamical investigation in (590.5)—but with a new significance, one related to *momentum*.

given point of the field. We called this vector the electromagnetic momentum at that point. This quantity may be considered as the time-integral of the electromotive intensity which would be produced at that point by the sudden removal of all the currents from the field. It is identical with the quantity already investigated in Art. 405 as the vector-potential of magnetic induction. Its components parallel to x, y, and z are F, G, and H. The electromagnetic momentum of a circuit is the line-integral of $\mathfrak{A}$ round the circuit.

604.2 We then, by means of Theorem IV, Art. 24, transformed the line-integral of $\mathfrak{A}$ into the surface-integral of another vector, $\mathfrak{B}$, whose components are $a, b, c,$ and we found that the phenomena of induction due to motion of a conductor, and those of electromagnetic force, can be expressed in terms of $\mathfrak{B}$. We gave to $\mathfrak{B}$ the name of the magnetic induction, since its properties are identical with those of the lines of magnetic induction as investigated by Faraday.

604.3 We also established three sets of equations: the first set, (A), are those of magnetic induction, expressing it in terms of the electromagnetic momentum. The second set, (B), are those of electromotive intensity, expressing it in terms of the motion of the conductor across the lines of magnetic induction, and of the rate of variation of the electromagnetic momentum. The third set, (C), are the equations of electromagnetic force, expressing it in terms of the current and the magnetic induction.

604.1, continued. *We called this vector the electromagnetic momentum at that point*: Although Maxwell does indeed use the term "electromagnetic momentum," he called it the *electrokinetic* momentum when he renamed vector $\mathfrak{A}$ in (590.5). In general, Maxwell tends to apply the two terms interchangeably—as, for example, at (579.3)

the electromotive intensity which would be produced … by the sudden removal of all the currents: Maxwell invoked such a thought experiment in (590.5). That it indeed indicates "something like momentum" is what he observed, in the name of Faraday, at (551.3). Compare his discussion of the role of *sudden stoppage* in the operation of the hydraulic ram in Art. 547.

The electromagnetic momentum … is the line-integral of $\mathfrak{A}$ *round the circuit*: Maxwell developed the line-integral of $\mathfrak{A}$ in Art. 405; he identified the electromagnetic (or electrokinetic) momentum of a circuit in a different form at (578.6).

604.3 *We also established three sets of equations*: Equations (A) were set forth in (591.5), equations (B) in (598.4), equations (C) in (603.1). They make up the first three of a total of twelve sets of General Equations of the Electromagnetic Field expounded in the present chapter.

604.4 The current in all these cases is to be understood as the actual current, which includes not only the current of conduction, but the current due to variation of the electric displacement.

604.5 The magnetic induction $\mathfrak{B}$ is the quantity which we have already considered in Art. 400. In an unmagnetized body it is identical with the force on a unit magnetic pole, but if the body is magnetized, either permanently or by induction, it is the force which would be exerted on a unit pole, if placed in a narrow crevasse in the body, the walls of which are perpendicular to the direction of magnetization. The components of $\mathfrak{B}$ are a, b, c.

604.6 It follows from the equations (A), by which a, b, c are defined, that

$$\frac{da}{dx}+\frac{db}{dy}+\frac{dc}{dz}=0.$$

604.7 This was shewn at Art. 403 to be a property of the magnetic induction.

605.] We have defined the magnetic force within a magnet, as distinguished from the magnetic induction, to be the force on a unit pole placed in a narrow crevasse cut parallel to the direction of magnetization. This quantity is denoted by $\mathfrak{H}$, and its components by α, β, γ. See Art. 398.

604.4 *the actual current, which includes not only … conduction, but … displacement*: Maxwell writes here as though he had already introduced the term "actual current" in the sense described. He indeed used that locution in (578.3); but since the discussion there was limited to electrical *circuits* (which are formed of conductors rather than dielectrics), no significant displacement could arise. Later in the present chapter, Arts. 610 and 611, Maxwell will use the term "true current" to denote the sum of conduction and displacement currents; but the present passage marks his first explicit mention of their summation.

604.6 *It follows from equations (A) … that* $\frac{da}{dx}+\frac{db}{dy}+\frac{dc}{dz}=0$: Equations (A) were given in (591.5) as $a=\frac{dH}{dy}-\frac{dG}{dz}$, $b=\frac{dF}{dz}-\frac{dH}{dx}$, $c=\frac{dG}{dx}-\frac{dF}{dy}$. Take the derivatives da/dx and db/dy; the terms in H cancel one another; similarly, the terms in F and G cancel pairwise when dc/dz is also taken. We performed a similar transformation in (24.2, *comment*).

This is actually an instance of a general theorem. Since a, b, c are the components of $\mathfrak{B}$, we recognize $\frac{da}{dx}+\frac{db}{dy}+\frac{dc}{dz}$ as $\nabla\bullet\mathfrak{B}$, the divergence of $\mathfrak{B}$ (25.9, *comment*). Likewise F, G, H being the components of $\mathfrak{A}$ (590.2), equations (A) are equivalent to $\mathfrak{B}=\nabla\times\mathfrak{A}$ or $\mathfrak{B}=\text{curl }\mathfrak{A}$ (591.5, *comment*); thus we have shown that div curl $\mathfrak{A}=0$. It will be evident that, by the same method, we can prove that the divergence of the curl of *any* vector is identically zero.

605.1 If $\mathfrak{I}$ is the intensity of magnetization, and A, B, C its components, then, by Art. 400,

$$\left.\begin{aligned} a &= \alpha + 4\pi A, \\ b &= \beta + 4\pi B, \\ c &= \gamma + 4\pi C. \end{aligned}\right\} \quad \text{(Equations of Magnetization)} \qquad \text{(D)}$$

605.2 We may call these the equations of magnetization, and they indicate that in the electromagnetic system the magnetic induction $\mathfrak{B}$, considered as a vector, is the sum, in the Hamiltonian sense, of two vectors, the magnetic force $\mathfrak{H}$, and the magnetization $\mathfrak{I}$ multiplied by 4π, or

$$\mathfrak{B} = \mathfrak{H} + 4\pi\mathfrak{I}.$$

In certain substances, the magnetization depends on the magnetic force, and this is expressed by the system of equations of induced magnetism given at Arts. 426 and 435.

606.] Up to this point of our investigation we have deduced everything from purely dynamical considerations, without any reference to quantitative experiments in electricity or magnetism. The only use we have made of experimental knowledge is to recognise, in the abstract quantities deduced from the theory, the concrete quantities discovered by experiment, and to denote them by names which indicate their physical relations rather than their mathematical generation.

606.1 In this way we have pointed out the existence of the electromagnetic momentum $\mathfrak{A}$ as a vector whose direction and magnitude vary from one part of space to another, and from this we have deduced, by a mathematical process, the magnetic induction, $\mathfrak{B}$, as a derived vector. We have not, however, obtained any data for determining either $\mathfrak{A}$ or $\mathfrak{B}$ from the

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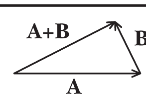
605.2 *the sum, in the Hamiltonian sense*: that is, the vector sum, the result of joining the tail of one vector to the head of the other, as indicated in the drawing.



606. *to recognize, in the abstract quantities deduced from theory, the concrete quantities discovered by experiment*: with its emphasis on "recognizing," this beautiful characterization makes it clear that experiment, for Maxwell, advances a doubly interpretive activity. By relating abstract mathematical quantities to experience we, first, enrich our experience with meaning and wholeness. Second, we *ground* our mathematical and symbolic knowledge, bestowing upon it a human face that Faraday, above all, would surely have prized: recall Maxwell's remark that "Faraday ... saw lines of force traversing all space where the mathematicians saw centres of force attracting at a distance" (Preface, paragraph 0.21).

denote them by names which indicate their physical relations rather than their mathematical generation: An expression of Maxwell's distaste for what Daniel M. Siegel has called "disembodied mathematics" in his richly rewarding study *Innovation in Maxwell's Electromagnetic Theory*, Cambridge University Press (1991), p. 31.

distribution of currents in the field. For this purpose we must find the mathematical connexion between these quantities and the currents.

606.2 We begin by admitting the existence of permanent magnets, the mutual action of which satisfies the principle of the conservation of energy. We make no assumption with respect to the laws of magnetic force except that which follows from this principle, namely, that the force acting on a magnetic pole must be capable of being derived from a potential.

606.3 We then observe the action between currents and magnets, and we find that a current acts on a magnet in a manner apparently the same as another magnet would act if its strength, form, and position were properly adjusted, and that the magnet acts on the current in the same way as another current. These observations need not be supposed to be accompanied by actual measurements of the forces. They are not therefore to be considered as furnishing numerical data, but are useful only in suggesting questions for our consideration.

606.4 The question these observations suggest is, whether the magnetic field produced by electric currents, as it is similar to that produced by permanent magnets in many respects, resembles it also in being related to a potential?

606.5 The evidence that an electric circuit produces, in the space surrounding it, magnetic effects precisely the same as those produced by a magnetic shell bounded by the circuit, has been stated in Arts. 482–485.

606.6 We know that in the case of the magnetic shell there is a potential, which has a determinate value for all points outside the substance of the shell, but that the values of the potential at two neighbouring points, on opposite sides of the shell, differ by a finite quantity.

606.7 If the magnetic field in the neighbourhood of an electric current resembles that in the neighbourhood of a magnetic shell, the magnetic potential, as found by a line-integration of the magnetic force, will be the same for any two lines of integration, provided one of these lines can be transformed into the other by continuous motion without cutting the electric current.

606.2 *the force acting on a magnetic pole must be capable of being derived from a potential*: Maxwell introduced derivation from a potential generally in (16.4–16.5). For the magnetic case see (395.1) and (398.1, *comment*).

606.3 *They are not ... to be considered as furnishing numerical data, but* [*as*] *suggesting questions*: He continues to point out the interpretive role of experiment.

606.6 *the values of the potential ... differ by a finite quantity*: We saw this in (411.).

606.7 *the magnetic potential ... will be the same for any two lines of integration, provided one of these lines can be transformed into the other...*: Maxwell introduced this idea in (481.), but not so explicitly as he states it here. See also (607.2) below.

606.8 If, however, one line of integration cannot be transformed into the other without cutting the current, the line-integral of the magnetic force along the one line will differ from that along the other by a quantity depending on the strength of the current. The magnetic potential due to an electric current is therefore a function having an infinite series of values with a common difference, the particular value depending on the course of the line of integration. Within the substance of the conductor, there is no such thing as a magnetic potential.

607.] Assuming that the magnetic action of a current has a magnetic potential of this kind, we proceed to express this result mathematically.

607.1 In the first place, the line-integral of the magnetic force round any closed curve is zero, provided the closed curve does not surround the electric current.

607.2 In the next place, if the current passes once, and only once, through the closed curve in the positive direction, the line-integral has a determinate value, which may be used as a measure of the strength of the current. For if the closed curve alters its form in any continuous manner without cutting the current, the line-integral will remain the same.

607.3 In electromagnetic measure, the line-integral of the magnetic force round a closed curve is numerically equal to the current through the closed curve multiplied by 4π.

607.4 If we take for the closed curve the rectangle whose sides are dy and dz, the line-integral of the magnetic force round the parallelogram is

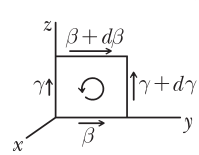
$$\left(\frac{d\gamma}{dy}-\frac{d\beta}{dz}\right)dy\,dz,$$

606.8 *Within … the conductor, there is no such thing as a magnetic potential*: Maxwell voiced an equivalent restriction in (485.1).

607.1 *the line-integral of the magnetic force round any closed curve is zero, provided the closed curve does not surround the electric current*: as stated earlier in (498.3).

607.3 *the line-integral of the magnetic force round a closed curve is numerically equal to the current through the closed curve multiplied by 4π*: To consider a simple case, the magnetic force $\mathfrak{H}$ at distance r from a long straight wire carrying current i is disposed circularly about the wire and has magnitude $T = 2i/r$, as stated at (477.1) and proved in (485.3, *comment*). For a circular path of radius r, the line-integral of $\mathfrak{H}$ will be $2\pi r \cdot 2i/r = 4\pi i$.

607.4 *the line-integral of the magnetic force round the parallelogram*: The drawing shows the parallelogram in question. I discussed a nearly identical case in the editor's introduction to Art. 24, section 2, on page 38 above. Proceeding in the same way, after collecting terms we shall find the line-integral equal to $d\gamma\,dz - d\beta\,dy$. Divide and multiply by $dy\,dz$ to obtain Maxwell's expression.



and if u, v, w are the components of the flow of electricity, the current through the parallelogram is $u\,dy\,dz$.

607.5 Multiplying this by 4π, and equating the result to the line-integral, we obtain the equation

$$\left.\begin{aligned} 4\pi u &= \frac{d\gamma}{dy} - \frac{d\beta}{dz}, \\ &\text{with the similar equations} \\ 4\pi v &= \frac{d\alpha}{dz} - \frac{d\gamma}{dx}, \\ 4\pi w &= \frac{d\beta}{dx} - \frac{d\alpha}{dy}, \end{aligned}\right\} \quad \begin{gathered}\text{(Equations of}\\ \text{Electric Currents)}\end{gathered} \qquad \text{(E)}$$

which determine the magnitude and direction of the electric currents when the magnetic force at every point is given.

607.6 When there is no current, these equations are equivalent to the condition that

$$\alpha dx + \beta dy + \gamma dz = -D\Omega,$$

or that the magnetic force is derivable from a magnetic potential in all points of the field where there are no currents.

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the current through the parallelogram is $u\,dy\,dz$: Strictly, u, v, and w are the components of the current *density*—current per unit cross section area. Only the component u is perpendicular to the yz-plane; thus the current through area $dy\,dz$ is $u\,dy\,dz$.

607.5 *Equations of Electric Currents* (E): Note that the right-hand sides of the equations together constitute curl $\mathfrak{H}$ or $\nabla\times\mathfrak{H}$; see Art. 25, especially (25.11 and *comment*). For α, β, γ are the components of $\mathfrak{H}$ (605.), while ∇ was defined as $\left(\mathbf{i}\frac{d}{dx}+\mathbf{j}\frac{d}{dy}+\mathbf{k}\frac{d}{dz}\right)$. Then $\nabla\times\mathfrak{H}=\left(\mathbf{i}\frac{d}{dx}+\mathbf{j}\frac{d}{dy}+\mathbf{k}\frac{d}{dz}\right)\times\left(\mathbf{i}\alpha+\mathbf{j}\beta+\mathbf{k}\gamma\right)$. Multiplying through, $\nabla\times\mathfrak{H}=\mathbf{i}\left(\frac{d\gamma}{dy}-\frac{d\beta}{dz}\right)+\mathbf{j}\left(\frac{d\alpha}{dz}-\frac{d\gamma}{dx}\right)+\mathbf{k}\left(\frac{d\beta}{dx}-\frac{d\alpha}{dy}\right)$. Since u, v, w have been identified as the components of current $\mathfrak{C}$ (603.2, 603.3), equations (E) may therefore be expressed in vector form as: $4\pi\mathfrak{C}=\nabla\times\mathfrak{H}$.

607.6 *When there is no current, ...* $\alpha\,dx+\beta dy+\gamma dz=-D\Omega$: D is Maxwell's symbol for an exact differential, introduced in (16.3). Then since α, β, γ are the components of $\mathfrak{H}$, the equation is equivalent to $\mathfrak{H}=-\nabla\Omega$; that is, $\mathfrak{H}$ is the negative gradient of Ω, which is therefore a *potential* (17.–17.4 and 17.5, *comment*). Now we just showed that equations (E) may be expressed as $4\pi\mathfrak{C}=\nabla\times\mathfrak{H}$; thus "when there is no current" they reduce to $\nabla\times\mathfrak{H}=0$ or curl $\mathfrak{H}=0$. But this will be identically true only if there is some scalar quantity Ω such that $\mathfrak{H}=-\nabla\Omega$; for you can easily prove that the curl of the gradient of any scalar quantity is zero. Thus "the magnetic force is derivable from a potential," Ω, wherever there are no currents.

607.7 By differentiating the equations (E) with respect to *x*, *y*, and *z* respectively, and adding the results, we obtain the equation

$$\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}=0,$$

which indicates that the current whose components are u, v, w is subject to the condition of motion of an incompressible fluid, and that it must necessarily flow in closed circuits.

607.8 This equation is true only if we take u, v, and w as the components of that electric flow which is due to the variation of electric displacement as well as to true conduction.

607.9 We have very little experimental evidence relating to the direct electromagnetic action of currents due to the variation of electric displacement in dielectrics, but the extreme difficulty of reconciling the laws of electromagnetism with the existence of electric currents which are not closed is one reason among many why we must admit the existence of transient currents due to the variation of displacement. Their importance will be seen when we come to the electromagnetic theory of light.

608.] We have now determined the relations of the principal quantities concerned in the phenomena discovered by Örsted, Ampère, and Faraday. To connect these with the phenomena described in the former parts of this treatise, some additional relations are necessary.

607.7 *By differentiating the equations (E)...*: Recall that Maxwell performed a similar transformation on equations (A) in (604.6). Here, equations (E) can be expressed as $4\pi\mathfrak{C}=\nabla\times\mathfrak{H}$; taking the divergence ($\nabla\bullet$) of both sides, we immediately deduce that $\nabla\bullet\mathfrak{C}=0$, since the divergence of $(\nabla\times\mathfrak{H})$ must be zero (604.6, *comment*). And, as before, we recognize $(du/dx+dv/dy+dw/dz)$ as *equivalent* to $\nabla\bullet\mathfrak{C}$.

the current whose components are u, v, w ... must necessarily flow in closed circuits: Otherwise stated, it has the solenoidal property (21.7 and *comment*).

607.8 *true only if we take u, v, w as ... due to the variation of electric displacement as well as to true conduction*: As we have noted already, the conduction current considered in itself is not in every case solenoidal. When charging a capacitor, for example, if we recognize *only* the conduction current, we shall say that electricity accumulates on the plates—a violation of the solenoidal condition (21.7). But when we also take the variation of displacement into account, we understand that electricity does not accumulate anywhere, not even on the plates (61.).

607.9 *the extreme difficulty of reconciling the laws of electromagnetism with ... electric currents that are not closed*: We can find a signal example in equations (E), expressed in vector form as $4\pi\mathfrak{C}=\text{curl}\,\mathfrak{H}$ (607.7, *comment*). As before, since the divergence of curl $\mathfrak{H}$ is identically zero, the divergence of $\mathfrak{C}$ must also be zero; but div $\mathfrak{C}$ will not be zero at the plates of a charging capacitor, like the one considered in the previous comment—unless we admit the existence of something equivalent to a current in the space between the plates.

the electromagnetic theory of light: to be put forward in Chapter XX.

608.1 When electromotive intensity acts on a material body, it produces in it two electrical effects, called by Faraday Induction and Conduction, the first being most conspicuous in dielectrics, and the second in conductors.

608.2 In this treatise, static electric induction is measured by what we have called the electric displacement, a directed quantity or vector which we have denoted by $\mathfrak{D}$, and its components by f, g, h.

608.3 In isotropic substances, the displacement is in the same direction as the electromotive intensity which produces it, and is proportional to it, at least for small values of this intensity. This may be expressed by the equation

$$\mathfrak{D} = \frac{1}{4\pi} K\mathfrak{E}, \quad \text{(Equation of Electric Displacement)} \qquad \text{(F)}$$

where K is the dielectric capacity of the substance. See Art. 68.

608.4 In substances which are not isotropic, the components f, g, h of the electric displacement $\mathfrak{D}$ are linear functions of the components P, Q, R of the electromotive intensity $\mathfrak{E}$.

608.5 The form of the equations of electric displacement is similar to that of the equations of conduction as given in Art. 298.

608.6 These relations may be expressed by saying that K is, in isotropic bodies, a scalar quantity, but in other bodies it is a linear and vector function, operating on the vector $\mathfrak{E}$.

609.] The other effect of electromotive intensity is conduction. The laws of conduction as the result of electromotive intensity were established by Ohm, and are explained in the second part of this treatise, Art. 241. They may be summed up in the equation

$$\mathfrak{K} = C\mathfrak{E}, \quad \text{(Equation of Conductivity)} \qquad \text{(G)}$$

where $\mathfrak{E}$ is the electromotive intensity at the point, $\mathfrak{K}$ is the density of the current of conduction, the components of which are p, q, and r, and C is the conductivity of the substance, which in the case of isotropic substances, is a simple scalar quantity, but in other substances becomes a linear and vector function operating on the vector $\mathfrak{E}$. The form of this function is given in Cartesian coordinates in Art. 298.

610.] One of the chief peculiarities of this treatise is the doctrine which it asserts, that the true electric current $\mathfrak{C}$, that on which the electromagnetic phenomena depend, is not the same thing as $\mathfrak{K}$, the current

608.5 *the equations of conduction ... given in Art. 298*: The Article is not included in the present selections, but Maxwell's point is that displacement and conduction can be viewed in comparable terms mathematically.

609. $\mathfrak{K}$ *is the density of the current of conduction*: that is, current per unit cross-section area of the conductor.

of conduction, but that the time-variation of $\mathfrak{D}$, the electric displacement, must be taken into account in estimating the total movement of electricity, so that we must write,

$$\mathfrak{C} = \mathfrak{K} + \dot{\mathfrak{D}}, \quad \text{(Equation of True Currents)} \qquad \text{(H)}$$

or, in terms of the components,

$$\left.\begin{aligned} u &= p + \frac{df}{dt}, \\ v &= q + \frac{dg}{dt}, \\ w &= r + \frac{dh}{dt}. \end{aligned}\right\} \qquad \text{(H*)}$$

611.] Since both $\mathfrak{K}$ and $\mathfrak{D}$ depend on the electromotive intensity $\mathfrak{E}$, we may express the true current $\mathfrak{C}$ in terms of the electromotive intensity, thus

$$\mathfrak{C} = \left(C + \frac{1}{4\pi} K \frac{d}{dt}\right)\mathfrak{E}, \qquad \text{(I)}$$

or, in the case in which C and K are constants,

$$\left.\begin{aligned} u &= CP + \frac{1}{4\pi} K \frac{dP}{dt}, \\ v &= CQ + \frac{1}{4\pi} K \frac{dQ}{dt}, \\ w &= CR + \frac{1}{4\pi} K \frac{dR}{dt}. \end{aligned}\right\} \qquad \text{(I*)}$$

612.] The volume-density of the free electricity at any point is found from the components of electric displacement by the equation

$$\rho = \frac{df}{dx} + \frac{dg}{dy} + \frac{dh}{dz}. \qquad \text{(J)}$$

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612. *free electricity*: Something of a figure of speech inherited from conventional fluid-flow thinking, the term was used in Art. 36 to denote electricity that is "free" to participate in conduction. It was there distinguished from "combined" or "bound" electricity, which undergoes displacement under the action of electromotive force but returns to its former condition as soon as that force is removed (60.2). But it is misleading to think in terms of *kinds* of electricities, or to think of *any* electricity as being free of force; Maxwell's discussions of conduction and induction have made it quite clear that the essential distinction is rather between forces of tension that easily break down (in a conductor) and those which endure (in a dielectric).

Equation (J): It is a variant of Poisson's equation (77.3). Since f, g, h are the components of displacement $\mathfrak{D}$, it can also be expressed as $\operatorname{div}\mathfrak{D} = \rho$ or, alternatively, as $\nabla \cdot \mathfrak{D} = \rho$ (77.2, *comment*).

613.] The surface-density of electricity is

$$\sigma = lf + mg + nh + l'f' + m'g' + n'h', \tag{K}$$

where l, m, n are the direction-cosines of the normal drawn from the surface into the medium in which f, g, h are the components of the displacement, and l', m', n' are those of the normal drawn from the surface into the medium in which they are f', g', h'.

614.] When the magnetization of the medium is entirely induced by the magnetic force acting on it, we may write the equation of induced magnetization,

$$\mathfrak{B} = \mu \mathfrak{H}, \tag{L}$$

where μ is the coefficient of magnetic permeability, which may be considered a scalar quantity, or a linear and vector function operating on $\mathfrak{H}$, according as the medium is isotropic or not.

615.] These may be regarded as the principal relations among the quantities we have been considering. They may be combined so as to eliminate some of these quantities, but our object at present is not to obtain compactness in the mathematical formulae, but to express every relation of which we have any knowledge. To eliminate a quantity which expresses a useful idea would be rather a loss than a gain in this stage of our enquiry.

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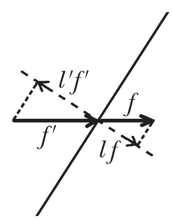
613. *the direction-cosines of the normal drawn from the surface into the medium*: Equation (K) gives the charge per unit area that develops on a surface between two different media in the presence of electric displacement. Since the normals are drawn away from the surface into their respective media, they are in opposite directions and their direction-cosines have opposite signs. The products $l'f'$ and lf will similarly have opposite signs. The sketch shows a boundary between two media; the positive x-direction is to the right. Since the net displacement in the direction of the normal, $lf - l'f'$, is directed to the left, the surface-density σ will be negative.



614. $\mathfrak{B} = \mu\mathfrak{H}$: Even though Maxwell has had the materials for writing this relation of magnetic proportionality since Part III Chapter IV, he has not done so until now, where it follows closely upon the electrical proportionality $\mathfrak{D} = \frac{1}{4\pi}K\mathfrak{E}$ (608.3). Is he inviting us to notice a parallelism between these relations? Certainly a formal correspondence is obvious. But physically, magnetism and electricity seem very far from being parallel. The nonexistence of isolated magnetic poles is a major point of difference; the lack of anything like a magnetic "current" is another.

615. *to express every relation of which we have any knowledge*: What impulse drives us to shape truth into speech? While it can have its comical side—"When I think, I must speak!" (Rosalind in *As You Like It*, Act III)—yet there is something quite wonderful about it, worthy even of the heavens: *The heavens declare the glory of God, and the firmament proclaims his handiwork. Day unto day utters speech, and night unto night expresses knowledge* (Psalm 19).