EDITOR'S INTRODUCTION

One of the most important of Maxwell's accomplishments in the *Treatise* was his mathematical integration of the seemingly disparate laws of electricity and magnetism into a single set of correlated equations. Maxwell made no effort to minimize the number of these equations, which in the *Treatise* total twenty in number. As he there wrote,

> [O]ur object at present is not to obtain compactness in the mathematical formulae, but to express every relation of which we have any knowledge. To eliminate a quantity which expresses a useful idea would be rather a loss than a gain in this stage of our enquiry. (615.)

Subsequently, these equations were reformulated by Oliver Heaviside as a beautiful, symmetrical, and logically compact set of four expressions utilizing the new *vector calculus*, which Heaviside himself did much to develop. Although they are universally called "Maxwell's equations" today, for many years these four vector expressions were termed the "Maxwell-Heaviside equations" in order to distinguish them from the equations Maxwell himself published. When written for the special case of empty space, they display a most remarkable symmetry:

$$\nabla \bullet \mu \mathbf{H} = 0$$
$$\nabla \bullet \varepsilon \mathbf{E} = 0$$
$$\nabla \times \mathbf{H} = \varepsilon \frac{d\mathbf{E}}{dt}$$
$$\nabla \times \mathbf{E} = -\mu \frac{d\mathbf{H}}{dt} .$$

At about the same time as Heaviside, but independently, Heinrich Hertz formulated an equivalent set of four fundamental equations in Cartesian coordinate form. First

$$\frac{dL}{dx} + \frac{dM}{dy} + \frac{dN}{dx} = 0 \qquad \frac{dX}{dx} + \frac{dY}{dy} + \frac{dZ}{dz} = 0 ,$$

which correspond to the first two of Heaviside's expressions, and next

$$\frac{1}{c}\frac{dL}{dt}=\frac{dY}{dz}-\frac{dZ}{dy} \qquad \frac{1}{c}\frac{dX}{dt}=\frac{dN}{dy}-\frac{dM}{dz}$$
$$\frac{1}{c}\frac{dM}{dt}=\frac{dZ}{dx}-\frac{dX}{dz} \qquad \frac{1}{c}\frac{dY}{dt}=\frac{dL}{dz}-\frac{dN}{dx}$$
$$\frac{1}{c}\frac{dN}{dt}=\frac{dX}{dy}-\frac{dY}{dx} \qquad \frac{1}{c}\frac{dZ}{dt}=\frac{dM}{dx}-\frac{dL}{dy},$$

which correspond to the second two.* Until at least the early twentieth century, it was customary to refer to Hertz's coordinate forms as the "Maxwell-Hertz equations," as Einstein did in his 1905 paper on Relativity.†

Maxwell had already articulated, in the *Treatise* and earlier writings, all that was necessary to derive the four equations in either vectorial‡ or coordinate form. It is the aim of this book to trace a path through the *Treatise* sufficient to arrive at Hertz's expressions. Accordingly, there is no need for the reader to know any vector calculus—although that very beautiful art is well worth knowing.§

Maxwell was primarily interested not in equations as such but in understanding. A truly Maxwellian path will therefore include his efforts to grapple with one of the most perplexing questions in physics: the ability of electric and magnetic forces to act *at a distance*.

When one electrified body attracts another, nothing obvious happens in the space between them; so it is easy to assume that the power exerted by one body simply *skips over* the intervening distance to act directly upon the other. But Michael Faraday had argued convincingly that such action cannot be direct, since it is dramatically altered when different materials occupy that distance. Even so, few theorists accepted Faraday's interpretation of electric action as a *state of tension* in the intervening material—still less his extension of that interpretation to *space itself* in cases where no ponderable matter was present. For how could "empty space" sustain tension?

To Maxwell, though, Faraday's researches clearly revealed that space had a material aspect. Maxwell would go on to identify space's ability to sustain electric tension as akin to *elasticity* and its magnetic susceptibility

* Hertz employed a left-handed coordinate system; I have rewritten his equations according to the modern right-handed system.

† "On the Electrodynamics of Moving Bodies" in Einstein *et al.*, *The Principle of Relativity*, Dover (1952).

‡ Maxwell would have used, and did use, quaternion rather than vector notation.

§ Interested readers may enjoy H. M. Schey's *Div, Grad, Curl and All That*, W. W. Norton Co. (1973).

as a kind of *inertia*. When Maxwell wishes to speak of space in its material character, he calls it *aether*.*

Deeply moved by Faraday's way of thinking (which employed no symbolic mathematics), Maxwell is not content to develop a formalistic theory. Instead he does his utmost to articulate *what happens in the medium* when it is subjected to electric or magnetic action. In the pursuit of understanding, Maxwell is committed to articulate speech, even when he has to use language that is figurative, metaphorical, or provisional. Accordingly, he is led to the concept of *electric displacement* (Art. 60) and to a distinctive, but today all-but-forgotten, treatment of *magnetic induction* (Art. 400).

A truly Maxwellian path to the equations will therefore include Maxwell's idiosyncratic approach to magnetism. Modern treatments invariably tackle magnetism as but a parallel to electrostatics: since both electrical charges and magnetic poles follow the same inverse-square law, it is easy to define magnetic quantities and construct magnetic theorems by simple analogy with the corresponding electrical ones. Mathematically, there can be no objection to such a procedure; but Maxwell clearly wished to develop magnetic theory on a physical basis, not as mere mathematical formalism. In Maxwell's day, conventional "action-at-a-distance" theory ascribed physicality to *electrical charges* and *magnetic poles*. These were conceived as discrete entities; and if their mutual action was altered by the presence of intervening materials (as Faraday had shown it to be), that was only—so convention taught—because the material itself contained such charges or poles. Maxwell meets this thinking with a rhetorical trope that occupies much of Part III of the *Treatise*: there he initially approaches both electric displacement and magnetic induction using the conventional language of physical charges and poles, but then systematically transforms that language into one that describes *processes in the electromagnetic medium*. These processes, he urges, are the truly physical entities; "charges" and "poles" are to be abandoned as mere hypothetical constructs which have outlived their purpose.

It may be useful here to add a few remarks on Maxwell's mathematical style. Mathematically savvy readers are sometimes dismayed by

* Maxwell is not always explicit about his usage of this term; quite often he employs a more literalistic language that seems to characterize aether as a separate substance (though a very strange one) that somehow inhabits all of space. Nevertheless, I suggest that he is far from committed to the view of aether as a substance in its own right and wishes at least to hold the question open.

Maxwell's frequent departure from practices that many of us have been taught to regard as mandatory. Whereas modern convention strives to represent each quantity consistently by one and the same symbol, Maxwell sometimes shifts from one symbol to another—often abruptly and without comment. Modern mathematicians employ the special symbol ∂ to distinguish partial differentiation from ordinary differention; but Maxwell makes no such distinction. Moreover, he is less than punctilious regarding the difference between "infinitesimal" and "very small."

None of these mannerisms betokens carelessness or sloppy thinking. In some cases, notably in partial differentiation, the formalism in question had not yet become standardized. But all reflect a more interesting motive: Maxwell never thought it a virtue to allow formalism to replace thought. The more our knowledge becomes enshrined in symbols, the more does our thinking become mechanical and routinized. Much like his admired predecessor Faraday, Maxwell regarded *prose* as the principal vehicle of thought.

The selections and commentary in this book are adapted from my considerably longer guide, *Maxwell's Treatise on Electricity and Magnetism: The Central Argument.** As in that work, I have here extended Maxwell's own numbering system from successive Articles to individual paragraphs. I have employed a conspicuous separator line on each page to distinguish Maxwell's text from my commentary. With the exception of the paragraph numbers just mentioned, material above the line faithfully reproduces at least one of the three editions of the *Treatise*; material below the line represents my effort to elucidate or interpret.

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* Green Lion Press (2014).