proportional to the times, and that their periodic times, the fixed stars being at rest, are in the sesquiplicate ratio of the distances from Saturn's center.

[The demonstration is omitted.]

Phenomenon 3

The five primary planets, Mercury, Venus, Mars, Jupiter, and Saturn, enclose the sun in their orbits.

That Mercury and Venus revolve around the sun is demonstrated from their lunar phases. When they shine with a full face, they are located beyond the sun, when halved they are even with the sun, and when sickle-shaped they are this side of the sun, sometimes passing across its disk in the manner of sunspots. Further, from Mars's full face near conjunction with the sun, and its gibbous face in the quadratures, it is certain that it encompasses the sun. The same is also demonstrated of Jupiter and Saturn from their ever full phases, for it is manifest from the shadows of satellites cast upon them that these shine with light borrowed from the sun.

Notes on Phenomenon 3

- The assertion of this Phenomenon does not include a claim that the earth moves around the sun. The sun, with its five encompassing planets, could revolve around the earth. This was the theory of the Danish astronomer Tycho Brahe (1546–1601).

- In order to think through the argument of this Phenomenon, we must understand how a planet would look given different angles between us observing from earth, the sun providing the light, and the planet.

Lemmita: How the Planet Looks

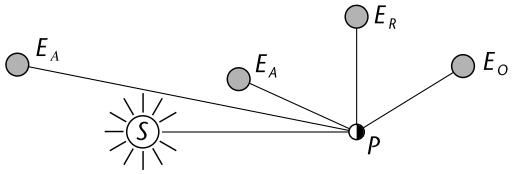
The following assumes that the light we see from the planets is reflected light of the sun. If we believed that the light might be coming from the planet, all this would change. In that case, however, it would be hard to make sense of the phase observations for Mercury, Venus and Mars. And Newton ends this Phenomenon with a demonstration that the light we see from Jupiter and Saturn is borrowed from the sun.

Given: Sun *S*, earth *E*, and planet *P*.

To Prove:

1. If angle *EPS* is right, the planet will show as half phase.
2. If angle *EPS* is acute, the planet will show as gibbous.
3. As angle *EPS* → 0, phase → full.
4. If angle *EPS* is obtuse, the planet will show as crescent.

Proof:

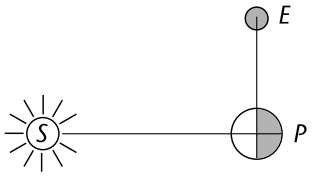


We will consider possibilities for any configuration of sun, earth, and planet.

We may have the earth at points E_A where the angle *EPS* is acute, either with the earth farther from the sun than the planet, or nearer.

Or, we may have the earth at a point such as E_R, at whatever distance, such that angle *EPS* is a right angle.

Finally, we may have the earth at a point such as E_O, at whatever distance, such that angle *EPS* is obtuse.

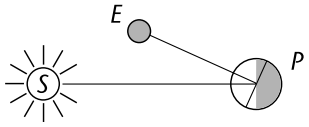


Case 1: First Consider Right Angle *EPS*

What do we see? (We must imagine this in three dimensions, and shift from the diagram's view from above the three bodies to a view from the surface of the earth.)

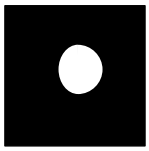
We would see half phase.

Case 2: Now Consider Acute Angle *EPS*

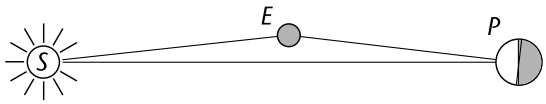


Adding the third dimension and shifting 90° to look at this from our earth's eye view, what do we see?

We would see gibbous.



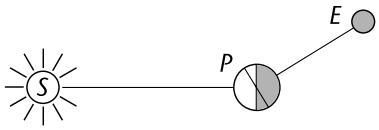
Case 3: What Happens when Acute Angle is Nearly 0°?



What would we see from the earth?

We would see the planet as nearly full.

Case 4: Now Consider an Obtuse Angle *EPS*



Shifting 90° into the paper to imagine this viewed from the earth, what would we see?

We would see a crescent.

Q.E.D.

Expansion of Newton's Sketch of Phenomenon 3

To Prove:

1. That Mercury and Venus revolve around the sun.

2. That Mars encompasses the sun.

3. That the same is demonstrated for Jupiter and Saturn.

Proof:

Part 1: Mercury and Venus

"That Mercury and Venus revolve around the sun is demonstrated from their lunar phases. When they shine with a full face, they are located beyond the sun, when halved they are even with the sun, and when sickle-shaped they are this side of the sun, sometimes passing across its disk in the manner of sunspots."

We must take as given the following additional observational conclusion Newton doesn't cite, but which was part of the common astronomical knowledge of the time:

General Observation:

The maximum elongation of Venus and Mercury is a small acute angle (48° for Venus, even less for Mercury). The elongation is the angle between the sun and the planet as we view it from the earth, angle *SEP* in these diagrams.

Thus we know for these two planets that angle *SEP* is always acute.

We also may consider the following particular observations, which Newton does cite:

Particular Observations:

Observation 1: We sometimes see Mercury and Venus near to full.

Observation 2: Sometimes we see them half full.

Observation 3: At other times they are seen as sickle-shaped (crescent).

Observation 4: Sometimes the planets are seen like spots traversing the sun's disk.

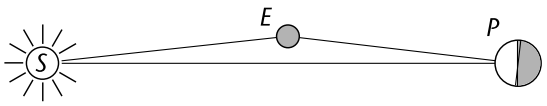
Step 1:

"That Mercury and Venus revolve around the sun is demonstrated from their lunar phases. When they shine with a full face, they are located beyond the sun…"

Now suppose by Observation 1 we find the planet near to full.

What must the configuration be then?

It could be close to full in this configuration:

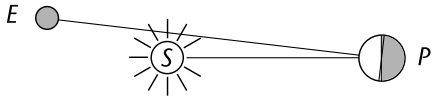


By the lemmita entitled How the Planet Looks, in the notes above, if phase is approaching full, angle *EPS* is approaching 0, as it does in this configuration.

But in this configuration angle *SEP* would not be small acute, contrary to the General Observation above.

Therefore this isn't the configuration.

It would also be near to full in this configuration:



Here angle $EPS \rightarrow 0$ as required by the lemmita. But here angle *SEP* also $\rightarrow 0$.

Therefore, when we see Mercury or Venus near to full, the planet is beyond the sun.

Q.E.D. for Step 1 of Part 1.

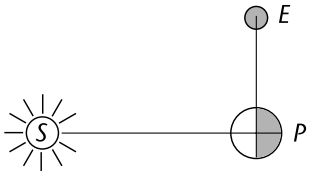
Step 2

"…when halved they are even with the sun, …"

By Observation 2, sometimes they are half full.

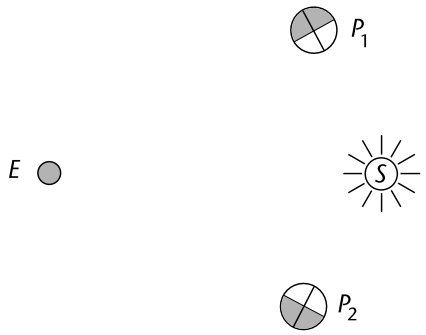
What configuration would give us this?

By the lemmita, angle *EPS* = a right angle.



The diagram below illustrates the two possible configurations, which are mirror images of one another. In both cases angle *SEP* is acute, consistent with the General Observation above.

Therefore, from the point of view of the earth, when the planet is half full, it is "even with the sun," to one side or the other, as shown:



Q.E.D. for Step 2 of Part 1.

Step 3a

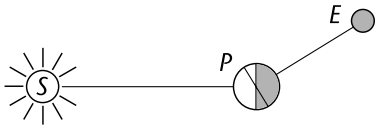
"…and when sickle-shaped they are this side of the sun,…"

By the lemmita, the planet would appear sickle-shaped or crescent when angle *EPS* is obtuse.

This can only happen in this configuration:

Here angle *SEP* is still acute, so it fits the General Observation.

Therefore when the planet is seen as crescent, it is on our side of the sun, between the sun and earth.



Q.E.D. for Step 3a of Part 1.

Step 3b

"…sometimes passing across its disk in the manner of sunspots."

By Observation 4, the planets sometimes are seen like spots traversing the sun's disk.

In this case again they must be passing between the sun and earth.

Q.E.D. for Step 3b of Part 1.

Conclusion of Part 1

Thus we have seen that Venus and Mercury are sometimes between us and the sun (Steps 3a and 3b), sometimes at the sides of the sun (Step 2) and sometimes beyond the sun (Step 1).

From this we conclude that Mercury and Venus revolve around the sun.

Q.E.D.

Part 2: Mars Encompasses the Sun

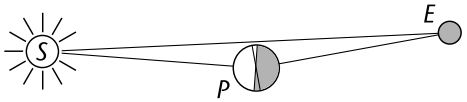
"Further, from Mars's full face near conjunction with the sun, and its gibbous face in the quadratures, it is certain that it encompasses the sun."

Step 1

"…full face near conjunction with the sun, …"

We observe that Mars is near full when it is near conjunction, that is, when it is on the same side of the earth as the sun. What configurations would put it on the same side?

Here is one configuration in which Mars and the sun are on the same side of the earth:

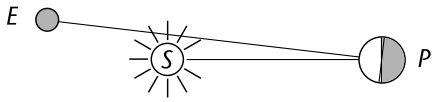


This is conjunction, but it wouldn't be full here, but "new." Angle *EPS* is

obtuse, and by the lemmita, the planet would be crescent.

In the configuration shown below, Mars and the sun are also on the same side.

Angle *EPS* is acute approaching 0°; therefore Mars's phase is approaching full.



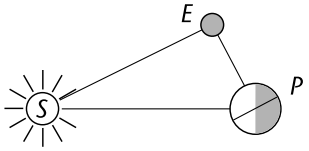
Therefore it must be this configuration; Mars is *beyond* the sun.

Q.E.D. for Step 1 of Part 2.

Step 2

"…and its gibbous face in the quadratures, …"

We observe also that when Mars is at quadrature, that is, when angle *SEP* = 90°, it is gibbous. By the lemmita, angle *EPS* would be acute.



From its quadrature configuration we know that distance *SP* opposite the right angle is the greatest side of the triangle and Mars is farther from the sun than the earth is from the sun.

Because Mars comes to quadrature, it is (at least at that point) farther from the sun than the earth is.

Q.E.D. for Step 2 of Part 2.

Conclusion of Part 2

"…it is certain that it encompasses the sun."

Thus we see that Mars is sometimes to the side of us (Step 2) and sometimes on the other side of the sun from us.

Therefore Mars's orbit encompasses the sun.

Q.E.D. for Part 2.

Part 3: Jupiter and Saturn

"The same is also demonstrated of Jupiter and Saturn from their ever full phases, …"

Our observation here is that they are full in all situations.

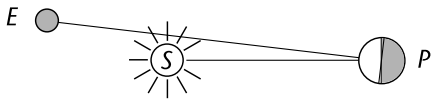
Consider conjunction, quadrature, and opposition.

Step 1: Conjunction

At conjunction Jupiter sets with the sun. There are two cases:

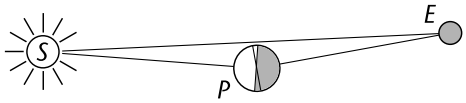
In the first case, angle *EPS* is small acute. By the lemmita, Jupiter could look full.

In the second case, angle *EPS* is obtuse. By the lemmita above, Jupiter wouldn't look full but crescent.



Thus if it looks full at conjunction, it must be in the first configuration.

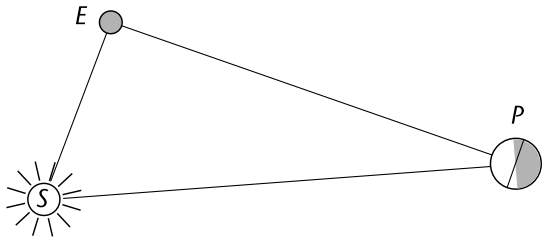
Therefore Jupiter must be beyond the sun at conjunction.



Q.E.D. for Step 1 of Part 3.

Step 2: Quadrature

At quadrature angle *SEP* = 90°. The observation is that in this configuration Jupiter looks full.



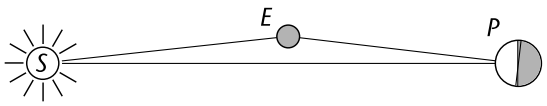
In Part 2, when Mars was at at quadrature, we said that it would appear gibbous. Jupiter, however, is so far away that it appears full even when angle $SEP = 90°$. This is because EP and SP are distances so great in relation to SE that angle EPS is small acute and so by the lemmita Jupiter would appear near to full. It is not visibly gibbous.

Q.E.D. for Step 2 of Part 3.

Step 3: Opposition

At opposition Jupiter rises as the sun sets.

Angle $EPS \rightarrow 0$ and the planet is full if it is on the opposite side of the earth from the sun.



(If Jupiter fell between earth and sun, angle EPS would be obtuse and we would not see the planet full.)

Q.E.D. for Step 3 of Part 3.

Conclusion of Part 3

Therefore, since they are sometimes beyond the sun and sometimes on our side of the sun, Jupiter and Saturn encompass the sun.

Q.E.D. for Part 3.

(See note at the end of Part 1.)

"…for it is manifest from the shadows of satellites cast upon them that these shine with light borrowed from the sun."

We see the moons of Jupiter and Saturn crossing the disks of the planets leaving shadows such that we can see both the moon and the shadow. This might be best seen at quadrature; the moon crosses into Jupiter before the shadow or vice versa.

This proves that Jupiter and Saturn are not their own light source.