FIRST DEFINITIONS

1. If from a point a straight line is joined to the circumference of a circle which is not in the same plane with the point, and the line is produced in both directions, and if, with the point remaining fixed, the straight line being rotated about the circumference of the circle returns to the same place from which it began, then the generated surface composed of the two surfaces lying vertically opposite one another, each of which increases indefinitely as the generating straight line is produced indefinitely, I call a conic surface, and I call the fixed point the vertex, and the straight line drawn from the vertex to the center of the circle I call the axis.

2. And the figure contained by the circle and by the conic surface between the vertex and the circumference of the circle I call a cone, and the point which is also the vertex of the surface I call the vertex of the cone, and the straight line drawn from the vertex to the center of the circle I call the axis, and the circle I call the base of the cone.

3. I call right cones those having axes perpendicular to their bases, and I call oblique those not having axes perpendicular to their bases.

4. Of any curved line which is in one plane, I call that straight line the diameter which, drawn from the curved line, bisects all straight lines drawn to this curved line parallel to some straight line; and I call the end of the diameter situated on the curved line the vertex of the curved line, and I say that each of these parallels is drawn ordinatewise to the diameter (τεταγμένως ἐπὶ τὴν διάμετρον κατῆχθαι).*

5. Likewise, of any two curved lines lying in one plane, I call that straight line the transverse diameter (διάμετρος πλαγία) which cuts the two curved lines and bisects all the straight lines drawn to either of the curved lines parallel to some straight line; and I call the ends of the [transverse] diameter situated on the curved lines the vertices of the curved lines; and I call that straight line the upright diameter (διάμετρος ὀρθία) which, lying between the two curved lines, bisects all the straight lines intercepted between the curved lines and drawn parallel to some straight line; and I say that each of the parallels is drawn ordinatewise to the [transverse or upright] diameter.

* We shall follow modern usage and generally call these parallels ordinates. (Tr.)

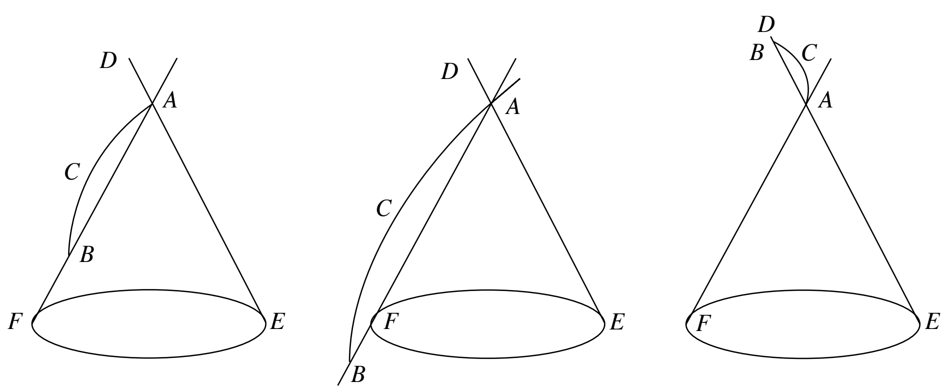
6. The two straight lines, each of which, being a diameter, bisects the straight lines parallel to the other, I call the conjugate diameters (συζυγεῖς διαμέτροι) of a curved line and of two curved lines.

7. And I call that straight line the axis of a curved line and of two curved lines which being a diameter of the curved line or lines cuts the parallel straight lines at right angles.

8. And I call those straight lines the conjugate axes of a curved line and of two curved lines which, being conjugate diameters, cut the straight lines parallel to each other at right angles.

PROPOSITION 1

The straight lines drawn from the vertex of the conic surface to points on the surface are on that surface.



Let there be a conic surface whose vertex is the point *A,* and let there be taken some point *B* on the conic surface, and let a straight line *ACB* be joined.

I say that the straight line *ACB* is on the conic surface.

For if possible, let it not be, and let the straight line *DE* be the line generating the surface, and *EF* be the circle along which *ED* is moved. Then if, the point *A* remaining fixed, the straight line *DE* is moved along the circumference of the circle *EF,* it will also go through the point *B* (Def. 1), and two straight lines will have the same ends. And this is absurd.

Therefore the straight line joined from *A* to *B* cannot not be on the surface. Therefore it is on the surface.

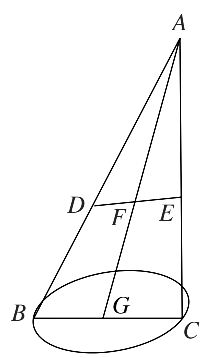
PORISM

It is also evident that, if a straight line is joined from the vertex to some point among those within the surface, it will fall within the conic surface; and if it is joined to some point among those without, it will be outside the surface.

PROPOSITION 2

If on either one of the two vertically opposite surfaces two points are taken, and the straight line joining the points, when produced, does not pass through the vertex, then it will fall within the surface, and produced it will fall outside.

Let there be a conic surface whose vertex is the point *A,* and a circle *BC* along whose circumference the generating straight line is moved, and let two points *D* and *E* be taken on either one of the two vertically opposite surfaces, and let the joining straight line *DE,* when produced, not pass through the point *A*.

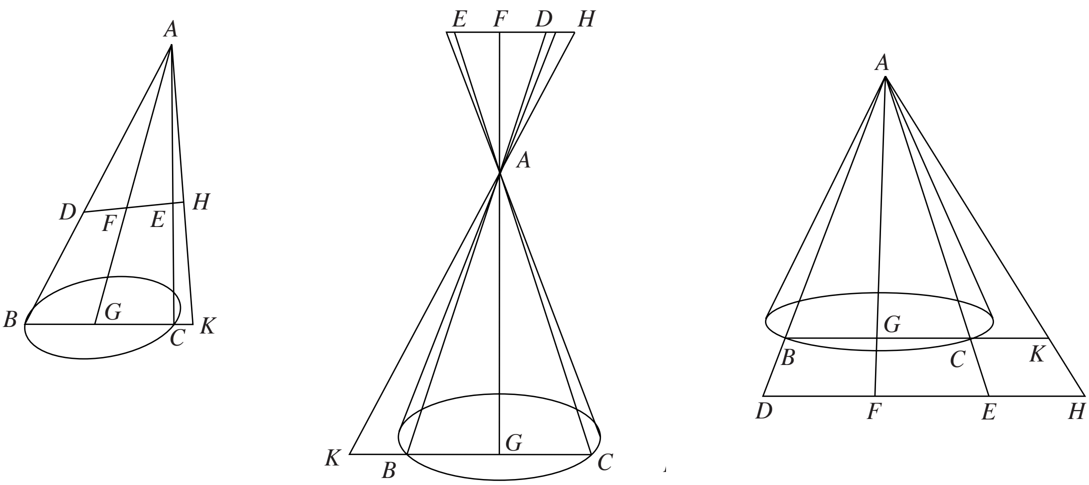


I say that the straight line *DE* will be within the surface, and produced will be without.

Let *AE* and *AD* be joined and produced. Then they will fall on the circumference of the circle (I. 1). Let them fall to the points *B* and *C,* and let *BC* be joined. Therefore the straight line *BC* will be within the circle, and so too within the conic surface.

Then let a point *F* be taken at random on *DE,* and let the straight line *AF* be joined and produced. Then it will fall on the straight line *BC*; for the triangle *BCA* is in one plane (Eucl. XI. 2). Let it fall to the point *G*. Since then the point *G* is within the conic surface, therefore the straight line *AG* is also within the conic surface (I. 1 porism), and so too the point *F* is within the conic surface. Then likewise it will be shown that all the points on the straight line *DE* are within the surface. Therefore the straight line *DE* is within the surface.

Then let *DE* be produced to *H*. I say then it will fall outside the conic surface.



For if possible, let there be some point *H* of it not outside the conic surface, and let *AH* be joined and produced. Then it will fall either on the circumference of the circle or within (I. 1 and porism). And this is impossible, for it falls on *BC* produced, as for example to the point *K*. Therefore the straight line *EH* is outside the surface.

Therefore the straight line *DE* is within the conic surface, and produced is outside.

PROPOSITION 3

If a cone is cut by a plane through the vertex, the section is a triangle.

Let there be a cone whose vertex is the point *A* and whose base is the circle